ANGULAR DISTRIBUTIONS OF HIGH-MASS DILEPTONS
WITH SOFT-GLUON EMISSION EFFECTS

M. HAYASHI
Department of Physics
(Received 5 January, 1982)

Abstract: The angular distributions for dileptons arising from decays of a virtual photon and the $Z^0$-boson produced in $pp$ collision are calculated at large mass $M$ and finite transverse momentum $Q_T$ of dileptons with $A \ll Q_T \ll M$, taking into account the soft-gluon emission effects in all orders of $\alpha_s$ (in double leading-logarithm approximation). Numerical evaluation for the differential cross-section $d\sigma/d\eta dy dQ_T$ and the coefficient of parity-violating term $\xi(M, Q_T)$ in the angular distribution is carried out for $pp$ collision at $\sqrt{s} = 540$ GeV and $y = 0$ in the limit $Q_T / M \rightarrow 0$ in the Gottfried-Jackson reference frame. We point out that the soft-gluon emission effects in the coefficient $\xi(M, Q_T)$ are deeply related to the scale-violation effects in the parton distribution.

§ 1. Introduction

The intermediate vector bosons $W^\pm$ and $Z^0$ of the standard $SU(2) \times U(1)$ model\(^1\) for electroweak interactions are expected to be discovered in the forthcoming experiments at CERN $\bar{p}p$ collider\(^2\). Much work has been devoted to the theoretical study for the various experimentally accessible quantities like dilepton-mass distribution\(^3\), single lepton spectra\(^4\), $x_F$ distribution of the single lepton\(^5\), various asymmetries (front-back asymmetry, charge asymmetry etc.) of the leptons\(^6\), etc. which can characterize the vector boson production in hadronic collisions via e.g.

$$\bar{p} + p \rightarrow (Z^0, \gamma^*) + X$$
$$\rightarrow l^+l^- (l = e, \mu, \tau)$$

using both the lowest-order quark-antiquark annihilation mechanism (Drell-Yan process) and the first order $O(\alpha_s)$ perturbative calculations of Quantum Chromodynamics (QCD).

In the previous paper\(^7\) we have studied the angular distributions of dileptons produced in hadronic collisions with large mass $M$ and finite transverse momentum $Q_T$ using the $O(\alpha_s)$ QCD calculations. However, the progress in the study of perturbative QCD\(^8\) has revealed that whereas the $O(\alpha_s)$ QCD calculations are reliable for the kinematical region $Q_T \sim O(M)$, in the kinematical region where two large mass scales $M$ and $Q_T$ are involved with $A \ll Q_T \ll M$, one has to take into account the important soft-gluon emission effects, i.e. sum all orders of perturbation theory in $\alpha_s^{0+1\ldots}$.

In the present paper we derive the formulas for the angular distributions of dileptons aris-
ing from the decay of the virtual photon $\gamma^*$ and the $Z^*$-boson produced via process (1.1) in the kinematical region with $A \ll Q_T \ll M$ where the soft-gluon effects play important role. We take into account the soft-gluon emission effects in terms of a quark form factor. The quark form factor of QCD was first derived by Dokshitzer, D'Yakonov and Troyan (DDT) in the double leading logarithm approximation (DLLA)\(^8\),\(^9\). Then Parisi and Petronzio\(^10\) have derived the improved formula for the quark form factor, pointing out that in the DLLA, the Sudakov-type (on mass shell, QED like) form factor should come out instead of the DDT-type form factor. This has been confirmed subsequently by several authors\(^12\)–\(^14\). Hence we adopt the Sudakov type quark form factor in this paper.

This paper is organized as follows: In Sect. 2 we present the formulas necessary for the numerical evaluation of the angular distributions of the dilepton produced in $p\bar{p}$ collision (1.1), taking into account the soft-gluon emission effects using the Sudakov-type quark form factor. In Sect. 3 we carry out numerical evaluation of the differential cross-section $d\sigma/dMdydQ_T^2|_{y=0}$ and the parity non-conserving coefficient of the angular distribution $\xi(M, Q_T)$ in the Gottfried-Jackson frame in the limit $Q_T/M \rightarrow 0$ for $p\bar{p}$ collision at $\sqrt{s}=540$ GeV. Sect. 4 is devoted to the concluding remarks.

§ 2. The formulas for the angular distributions of the lepton pair with soft-gluon emission effects

The formulas for the angular distributions of the lepton pair produced via hadronic reactions can be derived in an analogous way as in the case for the differential cross-section $d\sigma/dM^2dydQ_T^2$. In deriving such formulas the soft-gluon emission effects are taken into account by summing all orders of perturbation theory in $\alpha_s$, i.e. by calculating the diagrams of the type shown in Fig. 1 in DLLA. Referring the reader to Refs. 8) and 9) for the derivation of the formulas we present only the final formulas necessary for our further numerical evaluations. We give the formulas for the $Z^*$-boson production in $p\bar{p}$ collision (1.1). The angular distribution can be written in terms of the polar angle $\theta$ and the azimuthal angle $\phi$ of one of the leptons in the lepton pair rest system as\(^{16}\):

$$D(A_0, A_1, A_2, A_3, A_4) = 1 + \cos^2 \theta + A_0 \left(1 - \frac{\sin^2 \theta}{2}\right) + A_1 \sin 2\theta + A_2 \cos 2\theta,$$

$$+ A_3 \sin \theta \cos \phi + A_4 \cos \theta,$$

where the coefficients $A_i$ ($i=0, 1, \ldots, 4$) are functions of the total c.m. energy $\sqrt{s}$, $M$, $Q_T$ and rapidity $y$ of the lepton pair. They read as follows:

$$A_i \frac{d\sigma}{dM^2dydQ_T^2} = \frac{4\pi \alpha^2}{9} \frac{\partial}{\partial Q_T^2} \left(T^i(B(f)) \sum\right)$$

$$\{x_1 x_2 F_i(x_1, x_2, t) K_f(M)$$

$$\times \hat{A}_i (Q_T)\}, \quad i=0, 1, 2$$

$$+ \frac{4\pi \alpha^2}{9} \frac{\partial}{\partial Q_T^2} \left(T^i(B(f)) \sum\right)$$

$$\{x_1 x_2 G_i(x_1, x_2, t) L_f(M)$$

$$\times \hat{A}_i (Q_T)\}, \quad i=3, 4$$

\textit{Fig. 1 Emission of multiple gluons contributing to the production of $\gamma^*$ and $Z^*$ with transverse momentum $Q_T$.}
Angular Distributions of High-Mass Dileptons with Soft-Gluon Emission Effects

\[ \frac{da}{dM^2dydQ_T^2} = \frac{4\pi a_T^2}{9} \frac{\partial}{\partial Q_T^2} (T^2(B(t))) \sum_f \]

\[ x_1x_2F_f(x_1, x_3, t) K_f(M) , \]

(2.4)

where the Sudakov-type quark form factor

\[ T^2(B(t)) = \exp \left(-\frac{1}{2} B(t) \right), \]

\[ B(t) = \frac{4}{3} \alpha_s(t) \pi \ln^2(M^2/Q_T^2), \]

\[ t = \ln Q_T^2. \]

(2.4)

And

\[ \tilde{A}_0 = \tilde{A}_2 = \frac{p_{1x}^2 + p_{2x}^2}{|\vec{p}_1|^2 + |\vec{p}_2|^2}, \]

\[ \tilde{A}_1 = \frac{|\vec{p}_1|^2 + |\vec{p}_2|^2}{|\vec{p}_1|^2 + |\vec{p}_2|^2}, \]

\[ \tilde{A}_3 = \frac{|\vec{p}_1|^2 - |\vec{p}_2|^2}{|\vec{p}_1|^2 + |\vec{p}_2|^2}, \]

\[ \tilde{A}_4 = \frac{|\vec{p}_1|^2 - |\vec{p}_2|^2}{|\vec{p}_1|^2 + |\vec{p}_2|^2}. \]

(2.5)

Here the parton momenta \( \vec{p}_1 \) and \( \vec{p}_2 \) are expressed in terms of the incident (beam) momenta \( p^n \) and the target momenta \( p^p \) as: \( p_1 = x_1 p^n \) and \( p_2 = x_2 p^p \). We will present in §3 the numerical results for the angular distribution in the Gottfried-Jackson (GJ) frame in which the momenta of the incident and the hadrons are given by

\[ \vec{p}^n = \frac{\sqrt{x_T}}{4M} \exp \left(0, 0, 1 \right), \]

\[ \vec{p}^p = \frac{\sqrt{x_T}}{4M} \exp \left(-\frac{\sqrt{x_T}}{x_T^2}, 0, 0, \frac{x_T^2 - 4\pi}{x_T^2} \right) \]

with

\[ x_T = 2Q_T/\sqrt{s}, \quad \tau = M^2/s \] and \( \tilde{x_T} = \sqrt{x_T^2 + 4\pi} \).  

(2.6)

Furthermore, \( \sum_f \) denotes the summation over the quark flavours, and for \( pp \) collision (1. 1) we have:

\[ F_f(x_1, x_3, t) = f_f^p(x_1, t) f_f^p(x_3, t) \]

\[ + f_f^n(x_1, t) f_f^n(x_3, t) \]

\[ G_f(x_1, x_3, t) = f_f^p(x_1, t) f_f^n(x_3, t) \]

\[ - f_f^n(x_1, t) f_f^p(x_3, t) \]

(2.7)

\[ f_f^p(x_1, t) \text{ : a distribution function of } f \text{-quark in the incident (beam) hadron,} \]

\[ f_f^n(x_3, t) \text{ : a distribution function of the antiquark } \bar{f} \text{ in the target hadron}. \]

\[ K_f(M) = \frac{(a_f^2 + b_f^2)(a_f^2 + b_f^2)}{\sin^2\theta_w \cos^2\theta_w} |D_{x^2}|^2 \]

\[ - \frac{2e_f a_f}{\sin^2\theta_w \cos^2\theta_w} ReD_{x^2} \]

\[ L_f(M) = \frac{4a_f b_f}{\sin^2\theta_w \cos^2\theta_w} |D_{x^2}|^2 \]

\[ - \frac{2e_f b_f}{\sin^2\theta_w \cos^2\theta_w} ReD_{x^2} \]

(2.8)

\( a_f \) and \( b_f \) (a and b) are the vector and the axial vector coupling constants, in the units of e, of the f-quark (lepton) to the Z\(^0\)-boson and \( e_f \) is the charge of the f-quark divided by e and \( \theta_w \) is the Weinberg angle.

\[ a_f = -e_f \sin^2\theta_w + b_f, \]

\[ e_f = \frac{2}{3}, \quad b_f = \frac{1}{3} \quad \text{for } q_f = u, \text{c, or } t \]

\[ e_f = -\frac{1}{3}, \quad b_f = -\frac{1}{3} \quad \text{for } q_f = d, s, \text{or } b \]

\[ a = \sin^2\theta_w - \frac{1}{4}, \quad b = -\frac{1}{4} \quad \text{for } t = e, \mu, \text{or } \tau, \]

\[ a = \frac{1}{4}, \quad b = -\frac{1}{4} \quad \text{for } \nu_e, \nu_\mu, \text{or } \nu_\tau. \]

(2.9)

The propagator of the Z\(^0\) with its mass \( M_Z \) and the width \( \Gamma_Z \) is given by

\[ |D_{x^2}(M)|^2 \propto \left((M^2 - M_Z^2)^2 + M^2 \Gamma_Z^2\right)^{-1}, \]

\[ ReD_{x^2}(M) = (M^2 - M_Z^2)/D_{x^2}(M)^2. \]

(2.10)

Now let us consider the limit \( Q_T/M \to 0 \). Then the angular distributions for \( \bar{p}p \) collision are simplified significantly and they read in the GJ-frame as:

\[ D = \frac{16\pi}{3} \frac{dN}{dQ_T^2} = 1 + \cos^2\theta + \xi \cos \theta, \]

(2.11)

\[ \xi = \frac{\frac{\partial}{\partial Q_T^2} (T^2(B(t))) \sum_f G_f(x_1, x_3, t) L_f(M)}{\frac{\partial}{\partial Q_T^2} (T^2(B(t))) \sum_f T_f(x_1, x_3, t) K_f(M)} \]

(2.12a)

\[ \sum_f (G_f + \frac{\partial}{\partial Q_T^2} (G_f)/\frac{\partial}{\partial Q_T^2} (T^2)) L_f \]

\[ \sum_f (T_f + \frac{\partial}{\partial Q_T^2} (T_f)/\frac{\partial}{\partial Q_T^2} (T^2)) K_f, \]

(2.12b)

Note that the presence of the term contain-
ing $\xi$ reflects the intrinsic parity-violating nature of the $Z^0$-boson. For $M_\xi \ll M_Z$ we have $\xi \rightarrow 0$, hence we obtain $D \rightarrow 1 + \cos^2 \theta$, consistent with the well-known angular distribution of the Drell-Yan mechanism.

§ 3. Numerical results

We perform the numerical calculations for $\bar{p}p$ collision at $\sqrt{s} = 540$ GeV and $y = 0$, using Eqs (2.4) and (2.11). For the parton distributions we use those of Glück-Owens-Reya's (complete scale-violating) parametrization, calculated dynamically within the framework of QCD. They read as

$$F_{u,\text{sat}}(x, Q^2) = \sqrt{x} \left[ 5.707 (1-x)^3 - 6.219 \times (1-x)^5 + 4.570 (1-x)^7 - 2.868 (1-x)^9 \right],$$

$$F_{d,\text{sat}}(x, Q^2) = \sqrt{x} \left[ 2.994 (1-x)^4 - 0.787 \times (1-x)^5 + 1.890 (1-x)^7 - 3.026 (1-x)^9 \right],$$

$$F_{q,\text{sat}}(x, Q^2) = 0.019 (1-x)^{5.8} + 0.007 (1-x)^9 + 0.091 (1-x)^{13}$$

with $Q^2 = 3$. The $Q^2$ dependence in the range $2 \text{ GeV}^2 \leq Q^2 \leq 250$ GeV$^2$ is given by

$$F_{u,\text{sat}}(x, Q^2) = F_{u,\text{sat}}(x, Q_0^2) \times \left[ \ln \left( \frac{Q^2}{3 \times 10^{-6}} \right) \right] / \left( \ln (Q_0^2/3 \times 10^{-6}) \right),$$

$$F_{d,\text{sat}}(x, Q^2) = F_{d,\text{sat}}(x, Q_0^2) \times \left[ \ln \left( \frac{Q^2}{7 \times 10^{-7}} \right) \right] / \left( \ln (Q_0^2/7 \times 10^{-7}) \right),$$

$$F_{q,\text{sat}}(x, Q^2) = F_{q,\text{sat}}(x, Q_0^2) \times \left[ \ln \left( \frac{Q^2}{1 \times 10^{-5}} \right) \right] / \left( \ln (Q_0^2/1 \times 10^{-5}) \right).$$

We have fixed the number of flavours as $N_f = 6$ and the Weinberg angle as $\sin^2 \theta_W = 0.23$. Thus, we have used the values $M_Z = 91.56$ GeV which takes into account the radiative correction and $\Gamma_Z = 2.85$ GeV which is obtained in the Weinberg-Salam model by taking into account the $O(\alpha_s)$ QCD corrections.

For the running coupling constant we use

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f) \ln (Q^2/A^2)}.$$  

We present in Fig. 2 the differential cross-section $d\sigma/dMdydQ_T^2|_{y=0}$ as a function of $M$ for $\bar{p}p$ collision at $\sqrt{s} = 540$ GeV. Solid and dashed curves are the ones with the soft-gluon emission effects at $Q_T = 1.53 \times (10^9)$, 3, 5 and 10 GeV. Dotted curve is the $O(\alpha_s)$ result at $Q_T = 5$ GeV $\times (10^9)$.

In Fig. 3 $d\sigma/dMdydQ_T^2|_{y=0}$ at $M = 3$ is shown.
Angular Distributions of High-Mass Dileptons with Soft-Gluon Emission Effects

Fig. 3 The differential cross-section \( \frac{d\sigma}{dMdydQ^2} \bigg|_{y=0} \) as a function of \( Q_T \) for \( \bar{p}p \) collision at \( \sqrt{s} = 540 \text{ GeV} \), \( M = 30, 91.56 \text{ (} = M_Z \text{)} \) and 120 GeV.

30, 91.56 (\( = M_Z \)) and 120 GeV as a function of \( Q_T \).

We plot in Fig. 4 curves for \( \xi(M, Q_T) \) at \( Q_T = 1.35, 3, 5 \) and 10 GeV as a function of \( M \) (shown by solid line). The \( O(\alpha_s) \) result at \( Q_T = 5 \text{ GeV} \) is also shown by dotted line, for comparison.

Fig. 4 The coefficient of the parity-violating term \( \xi(M, Q_T) \) in the angular distribution for \( \bar{p}p \) collision at \( \sqrt{s} = 540 \text{ GeV} \), as a function of \( M \). Solid curves are the ones with the soft-gluon effects at \( Q_T = 1.35, 3, 5 \) and 10 GeV. Dotted curve is the \( O(\alpha_s) \) result at \( Q_T = 5 \text{ GeV} \) (\( \times \frac{1}{2} \)).

\[ 30, 91.56 = M_Z \text{ and 120 GeV as a function of } Q_T. \]

§ 4. Concluding remarks

We have derived the formulas for the angular distributions of lepton pairs decaying from the virtual photon \( \gamma^* \) and the \( Z^0 \)-boson produced in \( \bar{p}p \) collision at large \( M \) and finite \( Q_T \) with \( 1 \ll Q_T \ll M \) in the framework of the standard Weinberg-Salam model of electroweak interactions. The soft-gluon emission effects are taken into account by adopting the Sudakov-type quark form factor. We carried out the numerical evaluation of the differential cross-section \( \frac{d\sigma}{dMdydQ^2} \) for \( \bar{p}p \) collision at \( y = 0, \sqrt{s} = 540 \text{ GeV} \) and also the parity-violating coefficient \( \xi(M, Q_T) \) (see Eq (2.11)) of the angular distribution obtained in the GJ reference frame in the limit when \( Q_T / M \to 0. \)
We conclude that the soft-gluon emission effects play rather crucial role in the above-mentioned quantities for high-mass dileptons with $A \ll Q_T \ll M$.

Let us give several remarks.

1) In calculating the coefficient $\xi(M, Q_T)$ one should choose the scale in the (scale-violating) parton distributions as $Q^2 = Q_T^2$. However, as easily seen from Eq (2.12b), if one assumes $Q^2 = M^2$ formally or if one employs the scaling parton distributions one obtains:

$$\xi_{\text{soft gluon effects}} \to \xi_{\text{Drell–Yan}} = \frac{\sum G_f L_f}{\sum F_f K_f}.$$

(4.1)

Consequently one can argue that in the limit $Q_T/M \to 0$, the soft-gluon emission effects appearing in the parity non-conserving coefficient of the angular distribution are deeply connected with the scale-violating effects of the parton distributions with the scale $Q^2 = Q_T^2$.

2) For the sake of comparison, we have also carried out calculation using Altarelli-Parisi-Petronzio parametrization\(^{19}\) in which the effect of the scale-violation is incorporated in first order $O(\alpha_s)$ for the sea quark distribution function. In the kinematical region considered, the results are quite insensitive to the choice of parametrization.

3) The curves of $\xi(M, Q_T)$ exhibit characteristic structures, i.e. they change their sign just before the value of $M$ reaches $M_Z$. Such structure is also present in the case of $O(\alpha_s)$ calculation\(^{19}\).

4) Our discussion has been based on the Sudakov-type quark form factor. However it can be performed basing on other formalism. Work is now in progress to carry out evaluation using the more refined formalism\(^{12–14}\) in which the next-to-leading logarithmic contributions are also taken into account in the quark form factor of QCD. The results will be published elsewhere\(^{20}\).

References

8) For a present status of perturbative QCD, see e.g. M. Chaichian, M. Hayashi and T. Honkarante, Nucl. Phys. B175 (1980) 493.
Angular Distributions of High-Mass Dileptons with Soft-Gluon Emission Effects

(1979) 427.

18) For the extended gauge models of the electroweak interactions based on SU(2) × U(1) × SU(2)', SU(2) × U(1) × U(1)', etc, see e.g., M. Hayashi, S. Homma and K. Katsuura, submitted for publication in Ann. Phys. (N. Y.).